Experiments confirm that Gaussian smoothing of the objective function can be derived by an integral operation in image space with closed-form kernels. These blur kernels automatically turn out to be spatially varying (similar to human’s retina) for any non-displacement motion. Experiments confirm that Gaussian smoothing of the objective function outperforms Gaussian smoothing of images.

OBJECTIVE vs. SIGNAL SMOOTHING

Illustrative Example 1D Scale Alignment

We use 2-norm of signals difference as the alignment objective

\[
\int_a^b (f(x) - g(x))^2 \, dx
\]

Signal Smoothing

Objective Smoothing

EFFICIENT COMPUTATION OF SMOOTHED OBJECTIVE

Given a domain transformation \( \tau : X \times \Theta \to X' \), where \( X = \mathbb{R}^n \) and \( \Theta = \mathbb{R}^m \). Is there any \( \omega_{\tau} : X \times X' \to \mathbb{R} \) satisfying the following integral equation?

\[
\forall f, \quad \left[ f(\tau(x, \theta)) \ast k(\cdot ; \sigma^2) \right] (\theta) = \int_X f(y) \omega_{\tau}(\theta, x, y) \, dy
\]

CHECKING CORRECTNESS OF KERNELS

The alignment objective function:

\[
h(\theta) = \int_X f(\tau(x, \theta)) \, dx
\]

The smoothed alignment objective:

\[
z(\theta) = \int_X \left( f(\tau(x, \theta)) \ast k(\cdot ; \sigma^2) \right) \, dx
\]

1: Input: \( k : X \to \mathbb{R} \), \( \tau : X \times \Theta \to X' \), \( \Theta = \{ \theta_k \} \) for \( k = 1, \ldots, K \) s.t. \( \sigma_{k-1} < \sigma_k \)
2: for \( k = 1 \to K \) do
3: \( \theta_k = \) Local maximizer of \( z(\theta ; \sigma_k) \), initialized at \( \theta_{k-1} \)
4: end for
5: Output: \( \theta_K \)

EXPERIMENTS

Aligning the rectified view of each scene (shown below) against its perspective distorted copy.

- Horizontal Axis The degree of homography in effect, the larger, the more drastic the perspective.
- Vertical Axis Normalized correlation attained after algorithm’s convergence.

CLOSED FORM KERNELS FOR COMMON TRANSFORMATIONS

<table>
<thead>
<tr>
<th>Name</th>
<th>( \tau(x, \theta) )</th>
<th>( \omega_{\tau}(\theta, x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation</td>
<td>( x + d )</td>
<td>( \tau(x, \theta) ) ( y, \sigma^2 )</td>
</tr>
<tr>
<td>Translation-scale</td>
<td>( a \cdot x + b )</td>
<td>( \tau(\cdot ; \theta) ) ( y, \sigma^2 )</td>
</tr>
<tr>
<td>Affine</td>
<td>( A \cdot x + b )</td>
<td>( \tau(\cdot ; \theta) ) ( y, \sigma^2 )</td>
</tr>
<tr>
<td>Homography</td>
<td>( \frac{A \cdot x + b}{</td>
<td>A</td>
</tr>
</tbody>
</table>